A-0216

M. A. (Mathematics) Examination

April / May – 2015

P.D.E. & Fourier Analysis - 406

Time : 3 Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **M. A. (MATHEMATICS)**

Name of the Subject :

☛ **P.D.E. & FOURIER ANALYSIS : 406**

☛ Subject Code No. : **0 2 1 6** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) There are five questions in this question paper.
(3) Answer all questions.
(4) Figure to the right indicates full marks of the question.

Q1 A Obtain the direction ratio of tangent line to the curve in space. 7

B Show that the direction cosines of the tangent at the point (x, y, z) to the conic $ax^2 + by^2 + cz^2 = 1$ and x+y+z=1 are proportional to (by-cz, cz-ax, ax-by). 7

C Find the orthogonal trajectories on the sphere $x^2 + y^2 + z^2 = a^2$ of its intersections with the paraboloids xy=cz, being a parameter. 6

OR

Q1 A Prove that paraffin differential equation always possesses an integrating factor. 7

B Find the integral curves of the equations 7

$$\frac{dx}{x^2(y^3 - z^3)} = \frac{dy}{y^2(z^2 - x^3)} = \frac{dz}{z^2(x^3 - y^3)}.$$

C In usual notation discuss the method for solving the equation Pp+Qq=R, where P, Q, R are functions of x, y and z. 6

Q2 A State and prove the necessary and sufficient condition for the pfaffian differential equation to be integrable. 7

B Derive the condition for the partial differential equation f(x,y,z,p,q) = 0 and g(x,y,z,p,q) = 0 to be compatible. 7

C Explain Natani's method to solve the pfaffian differential equation. 6

OR

Q2 A Explain the method to obtain the orthogonal surfaces to the system of surface f(x, y, z) = c 7

B Explain Charpit's method to solve the partial differential equation f(x, y, z, p, q) = 0 7

C Solve the equation $(x^2z - y^3)dx + 3xy^2dy + x^3dz = 0$ if it is integrable. 6

Q3 A Show that the equations $xp - yq = x\,, x^2p + q = xz$ are compatible and find their solution. 7

B Prove that if $(\alpha_r D + \beta_{rD'} + \gamma_r)$ is a factor of F(D,D') and $d_r(\xi)$ is an arbitrary function of the single variable ξ then if $\alpha_r \neq 0$, $u_r = exp\left(-\frac{\gamma_r x}{\alpha_r}\right)\phi(\beta_r x - \alpha_r y)$ is a solution of the equation F(D,D')z=0. 7

C Show that the equation Z=px+qy is compatible to any equation f(x, y, z, p, q)=0 that is homogeneous in x, y and z and also solve the simultaneous equations , z=px+qy and 2xy$(p^2+q^2) = z(yp+xq)$. 6

OR

Q3 A Find the equation of the system of surfaces which cut orthogonally the cones of the system $x^2+y^2+z^2 = cxy$. 7

B Find the general solution of the partial differential equation r+s-2t=e^{x+y}. 7

C Prove that the equations f(x,y,p,q)=0 and g(x,y,p,q)=0 are compatible if $\frac{\partial(f,g)}{\partial(x,p)} + \frac{\partial(f,p)}{\partial(y,q)} = 0$. 6

Q4 A Find the Fourier series for the function $f(x) = x^2$, $-\pi < x < \pi$ and prove that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots\ldots = \frac{\pi^2}{12}$. 7

B Derive Fourier series formula for the interval $[0,2\pi]$. 7

C Prove that the sum of the squares of the fourier co-efficients of a square integrable function always converges. 6

OR

Q4 A Expand the function $f(x) = x - x^2, -1 < x < 1$ in terms of Fourier series. 7

B Prove that every orthogonal system of functions is linearly independent system. 7

C The expansion co-efficient of L^2-integrable function converges to zero as n is increased indefinitely for an orthogonal system. 6

Q5 A The temperature in semi-infinite rod which is bounded between $0 \le x < \infty$ is denoted by ' u ' and governed by the equation $u_t = k\, u_{xx}$ subject to the conditions $u = 0\ at\ t = 0\,; x \ge 0,\ \frac{\partial u}{\partial x} = -\mu\,; where\ \mu\ is\ constant\ when\ x = 0\ and\ t > 0.$ 7

$u(x,t)$ is bounded then show that $u(x,t) = \frac{2}{\pi}\mu \int_0^\infty \frac{\cos px}{p^2}\left(1 - e^{-p^2kt}\right)dp$.

B Find the fourier transform of the function $f(x) = \begin{cases} \frac{\sqrt{2\pi}}{2\varepsilon} & ; |x| \le \varepsilon \\ 0 & ; |x| > \varepsilon \end{cases}$. 7

C Find the fourier transform of the function $f(x) = \begin{cases} e^{iwx} & ; a < x < b \\ 0 & ; x < a, x > b \end{cases}$. 6

OR

Q5 A Solve the Heat equation $u_t = u_{xx}$ subject to the following condition 7

$u_x(0,t) = 0, u(x,0) = \begin{cases} x\ ; & 0 \le x < 1 \\ 0\ ; & x \ge 1 \end{cases}$ and $u(x,t)$ is bounded

B Find the fourier transform of the function $f(t) = \begin{cases} 1 - \frac{|t|}{a} & ; |t| < a \\ 0 & ; otherwise \end{cases}$ 7

C Find the fourier transform of the function $f(x) = \begin{cases} 1 & ; |x| < 1 \\ 0 & ; |x| > 1 \end{cases}$ 6

and hence evaluate $\int_0^\infty (\sin t)/t \quad dt$